Spin dynamics of a one-dimensional spin-1/2 fully anisotropic Ising-like antiferromagnet in a transverse magnetic field

This article has been downloaded from IOPscience. Please scroll down to see the full text article.
2001 J. Phys.: Condens. Matter 135205
(http://iopscience.iop.org/0953-8984/13/22/314)
View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.226
The article was downloaded on 16/05/2010 at 13:26

Please note that terms and conditions apply.

# Spin dynamics of a one-dimensional spin-1/2 fully anisotropic Ising-like antiferromagnet in a transverse magnetic field 

Asimkumar Ghosh<br>Department of Physics, Scottish Church College, 1 \& 3 Urquhart Square, Kolkata 700 006, India

Received 6 March 2001, in final form 19 April 2001


#### Abstract

We consider the one-dimensional Ising-like fully anisotropic $S=1 / 2$ Heisenberg antiferromagnetic Hamiltonian and study the dynamics of domain wall excitations in the presence of a transverse magnetic field $h_{x}$. We obtain dynamical spin correlation functions along the magnetic field, $S^{x x}(q, \omega)$, and perpendicular to it, $S^{y y}(q, \omega)$. It is shown that the line shapes of $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ are purely symmetric at the zone boundary. It is observed in $S^{y y}(q, \omega)$ for $\pi / 2<q<\pi$ that the spectral weight moves toward the lowenergy side with the increase of $h_{x}$. This model is applicable to the study of the spin dynamics of $\mathrm{CsCoCl}_{3}$ in the presence of weak interchain interactions.


## 1. Introduction

The spin- $\frac{1}{2}$ Ising-like antiferromagnetic (AFM) chain has been the subject of theoretical studies for quite some time. The spin dynamics of the system is characterized by a picture of propagating domain walls or solitons. The magnetic compounds $\mathrm{CsCoCl}_{3}$ and $\mathrm{CsCoBr}_{3}$ are good examples of $S=\frac{1}{2}$ Ising-like AFM chains. The simplest exchange interaction Hamiltonian describing these compounds is the $S=\frac{1}{2} X X Z$ Heisenberg model

$$
\begin{equation*}
H_{X X Z}=2 J \sum_{i}\left[S_{i}^{z} S_{i+1}^{z}+\epsilon\left(S_{i}^{x} S_{i+1}^{x}+S_{i}^{y} S_{i+1}^{y}\right)\right] \quad 0<\epsilon<1 \tag{1}
\end{equation*}
$$

For very small $\epsilon$, the lowest-order ground state of equation (1) is the Néel state with a $z$ component of the total spin given by $S_{\mathrm{T}}^{z}=0$. Villain [1] has calculated the longitudinal correlation function $S^{z z}(q, \omega)$ based on the basis states consisting of a single domain wall and predicted the appearance of a central peak with sharp shoulders. On the other hand, Ishimura and Shiba [2] proposed a picture of domain wall pair (DWP) states and showed that the propagating DWPs give rise to an excitation continuum around the Ising excitation energy $2 J$. The transverse correlation function $S^{x x}(q, \omega)$ exhibits a broad peak around $2 J$. The existence of these peaks of $S^{z z}(q, \omega)$ and $S^{x x}(q, \omega)$ has been verified by inelastic neutron
scattering experiments on $\mathrm{CsCoCl}_{3}[3-5]$ and $\mathrm{CsCoBr}_{3}$ [6]. A significant feature of the spinwave response of $S^{x x}(q, \omega)$ near the zone centre $(q=\pi)$ is that the spectral weights are heavily concentrated towards the lower-energy region. Nagler et al [6] added a staggered-field term

$$
\begin{equation*}
H_{\mathrm{S}}=h \sum_{i}(-1)^{i} S_{i}^{z} \tag{2}
\end{equation*}
$$

to the Hamiltonian in equation (1). The staggered field $h$ has two contributions, $h_{0}$ and $h_{\mathrm{ic}}$. The first contribution originates from taking account of the exchange mixing of higher levels with the ground doublet. The second contribution arises from the interchain exchange interactions at low temperatures. The interchain interactions treated in the mean-field approximation give rise to the staggered-field term $h_{\mathrm{ic}}$. The effective Hamiltonian contains both the terms $H_{X X Z}$ and $H_{\mathrm{S}}$. With this effective Hamiltonian, the broad peak is found to spilt into discrete peaks which is known as the Zeeman ladder and observed in Raman scattering on $\mathrm{CsCoCl}_{3}$ and $\mathrm{CsCoBr}_{3}$ [7]. However, the observed line shapes of $S^{x x}(q, \omega)$ are quite different from those of the theoretical predictions. Matsubara and Inawashiro [8] have included a weak next-nearest-neighbour (NNN) ferromagnetic (FM) interaction $H_{\mathrm{F}}$ in the Hamiltonian $H_{X X Z}$ in equation (1):

$$
\begin{equation*}
H_{\mathrm{F}}=-2 J^{\prime} \sum_{i}\left[S_{i}^{z} S_{i+2}^{z}+\epsilon\left(S_{i}^{x} S_{i+2}^{x}+S_{i}^{y} S_{i+2}^{y}\right)\right] \tag{3}
\end{equation*}
$$

They have shown the existence of bound states of DWPs as well as the free DWP states, and the transverse correlation function $S^{x x}(q, \omega)$ exhibits a sharp peak in the lower-energy region.

The effect of a transverse magnetic field $h_{x}$ on the spin dynamics of this model has been studied by Murao et al [9] who have shown that the spectral weight moves towards the low-energy side in $S^{y y}(q, \omega)$ for $\pi / 2<q<\pi$ with the increase in $h_{x}$, while there is no appreciable change in $S^{x x}(q, \omega)$ for all $q$. The distribution of intensities of the sharp peaks in $S^{y y}(q, \omega)$ varies irregularly for $q \approx \pi$. Although the proposed form of NNN FM coupling provides a good description of most of the experimental results, the required magnitude of the NNN exchange $\left|J^{\prime}\right| \sim 0.1|J|$ is unphysically large [10]. In 1996, Bose and Ghosh [11] proposed the Ising-like fully anisotropic Heisenberg AFM Hamiltonian in 1D and showed that the asymmetric line shapes of $S^{x x}(q, \omega)$ and the bound states of DWPs can be derived.

In the absence of the magnetic field, $S_{\mathrm{T}}^{z}$ is a good quantum number and the eigenvalues of different $S_{\mathrm{T}}^{z}$ having unequal numbers of DWPs form different energy bands separated by energy $2 J$. In the presence of a longitudinal magnetic field, $h_{z}, S_{\mathrm{T}}^{z}$ is still a good quantum number and the eigenvalues for the same value of $S_{\mathrm{T}}^{z}$ as well as the position of the peak of $S^{x x}(q, \omega)$ shift parallel with the increase in $h_{z}$. However, $S_{\mathrm{T}}^{z}$ is no longer a good quantum number in the presence of a transverse magnetic field $h_{x}$ and a mixing of states with different $S_{\mathrm{T}}^{z}$ occurs. Thus, eigenvalues as well as eigenstates will be modified by $h_{x}$ and the characteristics of the spin dynamics will be different.

In this paper, we study the effect of $h_{x}$ on the dynamical spin correlation functions in a fully anisotropic Ising-like $S=\frac{1}{2}$ Heisenberg AFM chain at low temperatures. Dynamical correlation functions $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ have been derived using the picture of propagating DWPs. Finally we introduce the use of this model to explain the spin dynamics of $\mathrm{CsCoCl}_{3}$ taking into account the weak interchain interactions (equation (2)). In section 2, the theory and the results for the eigenvalues of the DWP continuum and DWP bound states are derived. The dynamical spin correlation functions of $\mathrm{CsCoCl}_{3}$ are presented in section 3 . Section 4 contains a discussion of the results obtained.

## 2. Model and domain wall pair states

The one-dimensional fully anisotropic Ising-like Heisenberg Hamiltonian in the presence of a transverse magnetic field is given by

$$
\begin{align*}
H=2 \sum_{i}^{N} & {\left[J_{x} S_{i}^{x} S_{i+1}^{x}+J_{y} S_{i}^{y} S_{i+1}^{y}+J_{z} S_{i}^{z} S_{i+1}^{z}\right]-g_{\perp} \mu_{\mathrm{B}} H_{x} \sum_{i}^{N} S_{i}^{x} } \\
& =2 J \sum_{i}^{N}\left[S_{i}^{z} S_{i+1}^{z}+\frac{\epsilon_{1}}{2}\left(S_{i}^{+} S_{i+1}^{-}+S_{i}^{-} S_{i+1}^{+}\right)+\frac{\epsilon_{2}}{2}\left(S_{i}^{+} S_{i+1}^{+}+S_{i}^{-} S_{i+1}^{-}\right)\right]-h_{x} \sum_{i}^{N} S_{i}^{x} \tag{4}
\end{align*}
$$

where
$J=J_{z} \quad \epsilon_{1}=\frac{J_{x}+J_{y}}{2 J} \quad \epsilon_{2}=\frac{J_{x}-J_{y}}{2 J} \quad h_{x}=g_{\perp} \mu_{\mathrm{B}} H_{x} \quad \epsilon_{1}, \epsilon_{2} \ll 1$.
$H_{x}$ is the transverse magnetic field and we assume that $h_{x} \ll 2 J . N$ is the total number of spins. Since we are interested in excitations at low temperatures, we consider low-lying excited states. These states can be obtained from the Néel state by flipping a block of adjacent spins, giving rise to DWP states with $S_{\mathrm{T}}^{z}=0$ and $\pm 1$ (figure 1). These excitations occur around the Ising energy $2 J$ above the ground state. Following the method introduced by Murao et al [9], we classify these states into two series. Series $a$ starts from the state with $S_{\mathrm{T}}^{z}=1$ where two domain walls are adjacent. Let $m$ be the number (odd) of sites between two domain walls and $\phi_{1}^{(a)}(m)$ be the corresponding Ising state. The subsequent states $\phi_{j}^{(a)}(m)(j=2,3,4, \ldots)$ are generated from $\phi_{1}^{(a)}(m)$ such that the separation between the domain walls is increased by unit lattice distance successively towards the right-hand side of the chain. Hence,

$$
\begin{array}{lll}
\left.\phi_{1}^{(a)}(m)=S_{m}^{+} \mid \text {Néel }\right\rangle & & S_{\mathrm{T}}^{z}=1 \\
\phi_{j}^{(a)}(m)=S_{m+j-1}^{-} \phi_{j-1}^{(a)}(m) & (j=2,4,6, \ldots) & S_{\mathrm{T}}^{z}=0  \tag{5}\\
\phi_{j}^{(a)}(m)=S_{m+j-1}^{+} \phi_{j-1}^{(a)}(m) & (j=3,5,7, \ldots) & S_{\mathrm{T}}^{z}=1 .
\end{array}
$$

|Néel $\rangle$ is one of the Néel states. We choose a linear combination of these basis states for describing propagating DWPs with wave vector $q$ as

$$
\begin{equation*}
|j, q\rangle_{a}=\sqrt{\frac{2}{N}} \sum_{m=\text { odd }} \mathrm{e}^{-\mathrm{i} q m} \phi_{j}^{(a)}(m) \tag{6}
\end{equation*}
$$

On the other hand, series $b$ originates from the state with $S_{\mathrm{T}}^{z}=-1$, and the subsequent states with $S_{\mathrm{T}}^{z}=0$ and -1 appear alternately:

$$
\begin{array}{lll}
\left.\phi_{1}^{(b)}(m)=S_{m}^{-} \mid \text {Néel }\right\rangle & & S_{\mathrm{T}}^{z}=-1 \\
\phi_{j}^{(b)}(m)=S_{m+j-1}^{+} \phi_{j-1}^{(b)}(m) & (j=2,4,6, \ldots) & S_{\mathrm{T}}^{z}=0  \tag{7}\\
\phi_{j}^{(b)}(m)=S_{m+j-1}^{-} \phi_{j-1}^{(b)}(m) & (j=3,5,7, \ldots) & S_{\mathrm{T}}^{z}=-1 .
\end{array}
$$

Taking linear combination of these states with wave vector $q$,

$$
\begin{equation*}
|j, q\rangle_{b}=\sqrt{\frac{2}{N}} \sum_{m=\text { even }} \mathrm{e}^{-\mathrm{i} q m} \phi_{j}^{(b)}(m) . \tag{8}
\end{equation*}
$$



Figure 1. Néel states and DWP states for $S_{\mathrm{T}}^{z}= \pm 1$ and 0 . The dotted vertical lines indicate the positions of domain walls.

With the help of equations (5)-(8), one can obtain $H|j, q\rangle_{a}$ as follows:

$$
\begin{align*}
& H|1, q\rangle_{a}=2 J|1, q\rangle_{a}+V_{\epsilon_{1}}|3, q\rangle_{a}+V_{\epsilon_{2}}|1, q\rangle_{b}-\frac{h_{x}}{2}\left(|2, q\rangle_{a}+\mathrm{e}^{-\mathrm{i} q}|2, q\rangle_{b}\right) \\
& H|2, q\rangle_{a}=2 J|2, q\rangle_{a}+V_{\epsilon_{1}}|4, q\rangle_{a}-\frac{h_{x}}{2}\left(|1, q\rangle_{a}+|3, q\rangle_{a}+\mathrm{e}^{-\mathrm{i} q}|3, q\rangle_{b}+\mathrm{e}^{\mathrm{i} q}|1, q\rangle_{b}\right) \\
& \quad \vdots  \tag{9}\\
& \begin{aligned}
H|j, q\rangle_{a}= & 2 J|j, q\rangle_{a}+V_{\epsilon_{1}}|j+2, q\rangle_{a}+V_{\epsilon_{1}}^{*}|j-2, q\rangle_{a}-\frac{h_{x}}{2}\left(|j-1, q\rangle_{a}+|j+1, q\rangle_{a}\right) \\
& \quad-\frac{h_{x}}{2}\left(\mathrm{e}^{\mathrm{i} q}|j-1, q\rangle_{b}+\mathrm{e}^{-\mathrm{i} q}|j+1, q\rangle_{b}\right) \quad j \geqslant 3
\end{aligned}
\end{align*}
$$

where

$$
V_{\epsilon_{1}}=\epsilon_{1} J\left(1+\mathrm{e}^{-2 \mathrm{i} q}\right) \quad \text { and } \quad V_{\epsilon_{2}}=2 \epsilon_{2} J \cos q .
$$

In the same manner, one could derive a similar set of equations for $H|j, q\rangle_{b}$ in terms of $|n, q\rangle_{a}$ and $|n, q\rangle_{b}$. To avoid the mixing between the states of series $a$ and $b$, we further introduce symmetric ( $\alpha$ ) and antisymmetric ( $\beta$ ) functions [9] defined as

$$
\begin{align*}
|j, q\rangle_{\alpha} & =\frac{1}{\sqrt{2}}\left(|j, q\rangle_{a}+|j, q\rangle_{b}\right)  \tag{10}\\
|j, q\rangle_{\beta} & =\frac{1}{\sqrt{2}}\left(|j, q\rangle_{a}-|j, q\rangle_{b}\right)
\end{align*}
$$

Hence, one can express $H|j, q\rangle_{\alpha}$ as

$$
\begin{align*}
& H|1, q\rangle_{\alpha}=\left(2 J+V_{\epsilon_{2}}\right)|1, q\rangle_{\alpha}+V_{\epsilon_{1}}|3, q\rangle_{\alpha}-V_{\alpha}|2, q\rangle_{\alpha} \\
& H|2, q\rangle_{\alpha}=2 J|2, q\rangle_{\alpha}+V_{\epsilon_{1}}|4, q\rangle_{\alpha}-\left(V_{\alpha}^{*}|1, q\rangle_{\alpha}+V_{\alpha}|3, q\rangle_{\alpha}\right) \\
& \vdots  \tag{11}\\
& \begin{array}{r}
H|j, q\rangle_{\alpha}=2 J|j, q\rangle_{\alpha}+V_{\epsilon_{1}}|j+2, q\rangle_{\alpha}+V_{\epsilon_{1}}^{*}|j-2, q\rangle_{\alpha}-\left(V_{\alpha}^{*}|j-1, q\rangle_{\alpha}\right. \\
\left.\quad \quad+V_{\alpha}|j+1, q\rangle_{\alpha}\right) \quad j \geqslant 3
\end{array}
\end{align*}
$$

where

$$
V_{\alpha}=\frac{h_{x}}{2}\left(1+\mathrm{e}^{-\mathrm{i} q}\right) .
$$

Similarly, one can derive $H|j, q\rangle_{\beta}$ with $\alpha$ and $V_{\alpha}$ being replaced by $\beta$ and

$$
V_{\beta}=\frac{h_{x}}{2}\left(1-\mathrm{e}^{-\mathrm{i} q}\right)
$$

respectively. The first excited states can be constructed as a linear combination of symmetric and antisymmetric functions separately:

$$
\begin{equation*}
\Psi_{\alpha}(q)=\sum_{j} \alpha_{j}|j, q\rangle_{\alpha} \quad \text { and } \quad \Psi_{\beta}(q)=\sum_{j} \beta_{j}|j, q\rangle_{\beta} \tag{12}
\end{equation*}
$$

With the help of the equation (11), the following equations for the coefficients $\alpha_{j}$ and $\beta_{j}$ are obtained:

$$
\begin{align*}
& \lambda_{\alpha} \bar{\alpha}_{1}=\left(2 J+V_{\epsilon_{2}}\right) \bar{\alpha}_{1}+\bar{V}_{\alpha} \bar{\alpha}_{2}+\bar{V}_{\epsilon_{1}} \bar{\alpha}_{3} \\
& \lambda_{\alpha} \bar{\alpha}_{2}=2 J \bar{\alpha}_{2}+\bar{V}_{\alpha}\left(\bar{\alpha}_{1}+\bar{\alpha}_{3}\right)+\bar{V}_{\epsilon_{1}} \bar{\alpha}_{4} \\
& \vdots  \tag{13}\\
& \lambda_{\alpha} \bar{\alpha}_{j}=2 J \bar{\alpha}_{j}+\bar{V}_{\alpha}\left(\bar{\alpha}_{j-1}+\bar{\alpha}_{j+1}\right)+\bar{V}_{\epsilon_{1}}\left(\bar{\alpha}_{j-2}+\bar{\alpha}_{j+2}\right) \quad j \geqslant 3
\end{align*}
$$

where $\lambda_{\alpha}$ is the eigenvalue,

$$
\begin{aligned}
& \bar{V}=-h_{x} \cos \left(\frac{q}{2}\right) \\
& \bar{V}_{\epsilon_{1}}=2 \epsilon_{1} J \cos q
\end{aligned}
$$

and

$$
\bar{\alpha}_{j}=\alpha_{j} \mathrm{e}^{(\mathrm{i} q / 2) j}
$$

In the same manner, one can derive similar set of equations for $\beta$ with $\alpha$ being replaced by $\beta$, $\bar{V}_{\alpha}$ by

$$
\bar{V}_{\beta}=h_{x} \sin \left(\frac{q}{2}\right)
$$

and $\bar{\alpha}_{j}$ by

$$
\bar{\beta}_{j}=\beta_{j} \mathrm{e}^{(\mathrm{i}(k+\pi) / 2) j}
$$

Dispersion relations are obtained numerically by solving equations (13), with $N=1000$. Here, we present the results for $\epsilon_{1}=0.05$ and $\epsilon_{2}=0.10$, since these values are estimated for the compound $\mathrm{CsCoCl}_{3}$ [11]. Figure 2 shows the dispersion relations in the symmetric and antisymmetric modes, $\lambda_{\alpha}$ and $\lambda_{\beta}$, respectively, for $h_{x}=0$. The spin-wave continuum and the bound-state energy are plotted as solid and dotted lines, respectively. In the symmetric mode, the bound-state energy lies above the continuum for $q<\pi / 2$ and below the continuum for


Figure 2. The spin-wave excitation continuum (solid lines) and DWP bound-state energies (dotted lines) of the symmetric and antisymmetric modes for $\epsilon_{1}=0.05$ and $\epsilon_{2}=0.1$.
$q>\pi / 2$, while the reverse is true for the antisymmetric mode. The bound state does not exist when $\epsilon_{1}>\epsilon_{2}$. When $h_{x} \neq 0$, the energy band extends towards the high-energy region for $0 \leqslant q \leqslant \pi / 2$ in both the symmetric and the antisymmetric modes. The spin-wave excitations have a width at $q=\pi / 2$ in contrast with the case for $h_{x}=0$. The width also broadens with the increase of $h_{x}$. Note that the bound-state energy is not affected by the presence of $h_{x}$.

## 3. Dynamical spin correlation functions at $T=0 \mathrm{~K}$

The dynamical spin correlation function along the direction of $h_{x}$ at $T=0$ is defined as

$$
\begin{equation*}
\left.S^{x x}(q, \omega)=\sum_{\mathrm{e}}\left|\left\langle\Psi_{\mathrm{e}}\right| S^{x}(q)\right| \Psi_{\mathrm{g}}\right\rangle\left.\right|^{2} \delta\left(\omega-\lambda_{\mathrm{e}}+\lambda_{\mathrm{g}}\right) \tag{14}
\end{equation*}
$$

where $\left|\Psi_{g}\right\rangle,\left|\Psi_{e}\right\rangle$ denote the ground and excited states, respectively, and $\lambda_{\mathrm{g}}, \lambda_{\mathrm{e}}$ are the corresponding eigenvalues. In this case, the ground state is one of the Néel states and the summation extends over the first excited states only. Also

$$
S^{x}(q)=\frac{1}{2 \sqrt{N}} \sum_{j} \mathrm{e}^{\mathrm{i} q r_{j}}\left(S_{j}^{+}+S_{j}^{-}\right)
$$

Similarly, the dynamical spin correlation function perpendicular to the direction of $h_{x}$, $S^{y y}(q, \omega)$, is defined by replacing the superscript $x$ with $y$ in equation (14), where

$$
S^{y}(q)=\frac{1}{2 \mathrm{i} \sqrt{N}} \sum_{j} \mathrm{e}^{\mathrm{i} q r_{j}}\left(S_{j}^{+}-S_{j}^{-}\right)
$$

Since the ground state is the Néel state, $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ directly reflect the wavenumber dependence of the excited states.

With the help of the equation (12), the dynamical spin correlation functions can further be written as [9]

$$
\begin{align*}
& S^{x x}(q, \omega)=\frac{1}{4} \sum_{\alpha}\left|\alpha_{1}\right|^{2} \delta\left(\omega-\lambda_{\alpha}+\lambda_{\mathrm{g}}\right) \\
& S^{y y}(q, \omega)=\frac{1}{4} \sum_{\beta}\left|\beta_{1}\right|^{2} \delta\left(\omega-\lambda_{\beta}+\lambda_{\mathrm{g}}\right) \tag{15}
\end{align*}
$$

Note that $S^{x x}(q, \omega)$ depends only depends on $\left|\alpha_{1}\right|^{2}$ while $S^{y y}(q, \omega)$ depends on $\left|\beta_{1}\right|^{2}$. Thus, the symmetric mode is directly reflected in $S^{x x}(q, \omega)$, whereas the antisymmetric mode is reflected in $S^{y y}(q, \omega)$. The functions $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ for $h_{x}=0$ are shown in figure 3. A sharp peak originates from the bound state, whereas the broad peak originates from the free DWP states. The intensity of the sharp peak does not depend on the number of spins $N$, while the broad peak comprises $N-1$ peaks which have intensity of the order of $1 / N$. Note that at the zone boundary $(q=\pi / 2)$, the width of the continuum vanishes. This is also verified in neutron scattering experiments on $\mathrm{CsCoCl}_{3}$ [12].

The line shapes of $S^{x x}(q, \omega)$ at $h_{x}=0.1 J$ have been plotted in figure 4 . The main feature of $S^{x x}(q, \omega)$ induced by $h_{x}$ shows that the line shape is purely symmetric at the zone boundary


Figure 3. The functions $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ for $q=0, \pi / 2$ and $\pi$ in histograms with $\Delta \omega=0.025 \mathrm{~J}$.


Figure 4. The function $S^{x x}(q, \omega)$ for different values of $q$ and $h_{x}=0.1 J$. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.
and it is highly asymmetric away from the zone boundary. At $q \approx 0$, the sharp peak occurs at high energy and the tail in the lower-energy region. For $0<q<\pi / 2$, the spectral weight concentrates mainly in the middle of the continuum. The sharp peak emerges again in the lower-energy region for $\pi / 2<q<\pi$. At $q \approx \pi$, the line shape is not affected by $h_{x}$ as expected from the dispersion relation shown in figure 2.

Figure 5 shows $S^{y y}(q, \omega)$ at $h_{x}=0.1 J$. The line shape of $S^{y y}(q, \omega)$ is again symmetric at $q=\pi / 2$. The $q$-dependence of $S^{y y}(q, \omega)$ is opposite to that of $S^{x x}(q, \omega)$ and it remains


Figure 5. The function $S^{y y}(q, \omega)$ for different values of $q$ and $h_{x}=0.1 J$. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.
unaffected at $q=0$. For $0<q<\pi / 2$, the sharp peak appears in the lower-energy region of the broad peak. The sharp peak originates from the bound state, whereas the broad peak originates from the DWP continuum. For $\pi / 2<q<\pi$, the sharp peak appears on the higher-energy side of the broad peak and the tail is found to be enhanced towards the lowerenergy region.

In figures 6 and $7, S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ are shown for different values of $h_{x}$, respectively. With the increase of $h_{x}$, the features mentioned above are enhanced. The


Figure 6. The function $S^{x x}(q, \omega)$ for different values of $q$ and $h_{x}$. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.
height of the sharp peak is found to diminish with the increase of $h_{x}$. Note that $S^{x x}(q, \omega)$ for $0<q<\pi / 2$ and $S^{y y}(q, \omega)$ for $\pi / 2<q<\pi$ are sensitive to $h_{x}$ as observed by Murao et al [9].

### 3.1. Interchain interactions in $\mathrm{CsCoCl}_{3}$

Now we add the staggered-field term to the Hamiltonian in equation (5). The full Hamiltonian looks as follows:

$$
\begin{gather*}
H=2 J \sum_{i}^{N}\left[S_{i}^{z} S_{i+1}^{z}+\frac{\epsilon_{1}}{2}\left(S_{i}^{+} S_{i+1}^{-}+S_{i}^{-} S_{i+1}^{+}\right)+\frac{\epsilon_{2}}{2}\left(S_{i}^{+} S_{i+1}^{+}+S_{i}^{-} S_{i+1}^{-}\right)\right] \\
-h_{x} \sum_{i}^{N} S_{i}^{x}-h_{\text {ic }} \sum_{i}^{N}(-1)^{i} S_{i}^{z} . \tag{16}
\end{gather*}
$$

Here, we further consider that the staggered field $h_{\text {ic }}$ originates due to the weak interchain interaction. The interchain interaction has been treated in the mean-field approximation. Thus,

$$
\begin{equation*}
h_{\mathrm{ic}}=2 J^{\prime} \sum_{\delta}\left\langle S_{i+\delta}^{z}\right\rangle \tag{17}
\end{equation*}
$$



Figure 7. The function $S^{y y}(q, \omega)$ for different values of $q$ and $h_{x}$. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.
where $J^{\prime}$ is the interchain interaction strength and $\delta$ is the nearest neighbour on the $a b$-plane. Following the same technique as was developed in section 2, we obtain $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ numerically.

In figure $8, S^{x x}(q, \omega)$ is shown for different values of $h_{\text {ic }}$. When $h_{\text {ic }} \neq 0$, the broad peak of $S^{x x}(q, \omega)$ has been split into discrete peaks, which are known as the Zeeman ladder [13]. With the increase of $h_{\text {ic }}$, the number of peaks decreases and the separation between them becomes wider. For $\pi / 2<q<\pi$, the intensity of the peak at the lowest energy is stronger. Several peaks having nearly the same intensity are observed at $q \approx 0$.

Figure 9 shows $S^{y y}(q, \omega)$ for different values of $h_{\mathrm{ic}}$. For $0 \leqslant q \leqslant \pi / 2$, several strong peaks appear in the lower-energy region. Note that the differences between $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ are remarkable at $q=\pi$. In $S^{y y}(q, \omega)$ for $q=\pi$, several strong peaks appear in the higher-energy region and weak peaks in the lower-energy region, and their intensities vary irregularly with the increase in $h_{\mathrm{ic}}$. Thus the combined effect of $h_{x}$ and the interchain interactions is also found in $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ for $q \geqslant \pi / 2$ as reported in reference [9].

In $\mathrm{CsCoCl}_{3}$, the magnetic $\mathrm{Co}^{2+}$ ions are surrounded by trigonally distorted octahedra of $\mathrm{Cl}^{-}$ions and form chains along the $c$-axis with successive octahedra sharing a common face. The $\mathrm{CsCl}_{3}^{-}$chains are arranged in a triangular array. Since the exchange coupling between chains is antiferromagnetic, the triangular array forms a frustrated system. Thus there is no possibility of a perfectly regular AFM ordered state. Two different three-dimensionally ordered phases occur in $\mathrm{CsCoCl}_{3}$. First, below $T_{\mathrm{N}_{1}} \sim 21 \mathrm{~K}$ a partially disordered AFM phase (A) is


Figure 8. The function $S^{x x}(q, \omega)$ for different values of $h_{\text {ic }}$ and $h_{x}=0.1 J$. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.
formed in which a third of the chains are paramagnetic. A phase change takes place below $T_{\mathrm{N}_{2}} \sim 10-14 \mathrm{~K}$, to a ferrimagnetic phase ( F ), in which the paramagnetic chains align in the same direction, with the result that two thirds of the chains are aligned in one direction and one third in the opposite direction (figure 10) [14].

It is obvious from equation (17) that the staggered field $h_{\text {ic }}$ could take one of the two possible values $h_{\mathrm{ic}}=6 J^{\prime}$ and 0 . Similarly, in the partially disordered phase, $h_{\mathrm{ic}}$ takes one of the four values $h_{\mathrm{ic}}=6 J^{\prime}, 4 J^{\prime}, 2 J^{\prime}$ and 0 . The functions $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ of $\mathrm{CsCoCl}_{3}$ are obtained by summing those of the individual chains. In the ferrimagnetic phase, the ratio of the numbers of chains with $h_{\mathrm{ic}}=6 J^{\prime}$ and 0 is estimated as $1: 2$. In the partially disordered phase, the ratio of the numbers of chains with $h_{\text {ic }}=6 J^{\prime}, 4 J^{\prime}, 2 J^{\prime}$ and 0 is estimated as 1:3:3:5 [9]. Figure 11 shows $S^{x x}(q, \omega)$ for $q=0$ and $S^{y y}(q, \omega)$ for $q=\pi$ in the ferrimagnetic phase. The line shape of $S^{x x}(q, \omega)$ is affected strongly by $h_{x}$. As $h_{x}$ increases, the differences in intensities among various discrete peaks reduce markedly and the sharp peak disappears. On the other hand, $S^{y y}(q, \omega)$ is affected only slightly by $h_{x}$. The spectral weight is enhanced towards the lower-energy region with the increase in $h_{x}$, and the intensity distribution varies irregularly. $S^{x x}(q, \omega)$ for $q=0$ and $S^{y y}(q, \omega)$ for $q=\pi$ obtained in the partially disordered phase are shown in figure 12. $S^{x x}(q, \omega)$ is strongly affected by $h_{x}$ while $S^{y y}(q, \omega)$ is much less affected. $S^{x x}(q, \omega)$ for $q=\pi$ and $S^{y y}(q, \omega)$ for $q=0$ are not affected by $h_{x}$ in either the ferrimagnetic or the partially disordered phases.


Figure 9. The function $S^{y y}(q, \omega)$ for different values of $h_{\text {ic }}$ and $h_{x}=0.1 J$. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.


Figure 10. Magnetic ordering in the $a b$-plane below $T_{\mathrm{N}_{1}}$ : (a) ferrimagnetic structure for $T<T_{\mathrm{N}_{2}}$ (F phase); (b) structure for $T_{\mathrm{N}_{2}}<T<T_{\mathrm{N}_{1}}$ (A phase). Chains marked with open circles are disordered.

## 4. Discussion of results

We have studied the effect of the transverse magnetic field $h_{x}$ on dynamical properties of the one-dimensional fully anisotropic Ising-like antiferromagnet at low temperatures. We have shown using this Hamiltonian that some of the results obtained by Murao et al in [9], where a FM NNN interaction is assumed besides the usual AFM NN interaction, can be qualitatively reproduced. These include the formation of DWP bound states, two types of excited mode which are symmetric and antisymmetric with respect to the states with $S_{\mathrm{T}}^{z}=1$ and -1 and


Figure 11. The function $S^{x x}(q, \omega)$ for $q=0$ and $S^{y y}(q, \omega)$ for $q=\pi$ in the ferrimagnetic phase. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.
an asymmetry in the line shapes of the correlation functions $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$. In order to obtain the asymmetry in the line shapes of $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$, a FM NNN exchange of magnitude $\left|J^{\prime}\right| \sim 0.1|J|$ is required, which, considering that the NNN exchange is through two nonmagnetic ligands, would seem to be unphysically large [10]. On the other hand, our model could explain all of these characteristics with the usual NN AFM exchange interactions. There are, however, a number of differences. Murao et al [9] observed a single bound-state branch which is symmetric with respect to the zone boundary, whereas in the present study, different bound-state branches are obtained for symmetric and antisymmetric


Figure 12. The function $S^{x x}(q, \omega)$ for $q=0$ and $S^{y y}(q, \omega)$ for $q=\pi$ in the partially disordered phase. The width of each histogram is $\Delta \omega=0.025 \mathrm{~J}$.
modes which are asymmetric with respect to the zone boundary. No experimental evidence is as yet available on the effect of bound states on the thermodynamic and dynamic properties of the compounds $\mathrm{CsCoCl}_{3}$ and $\mathrm{CsCoBr}_{3}$. The symmetric modes contribute to $S^{x x}(q, \omega)$, whereas antisymmetric modes contribute to $S^{y y}(q, \omega)$. Both $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ have symmetry at the zone boundary even in the presence of either (i) both $h_{x}$ and $h_{\text {ic }}$ or (ii) either one of them. This symmetry is totally lost away from the zone boundary. This theory is also valid for analysis of the spin dynamics of $\mathrm{CsCoCl}_{3}$ and $\mathrm{CsCoBr}_{3}$ at finite temperatures, because
the correlation length of the spin along the chain is very large even in the paramagnetic phase ( $T \approx 21 \mathrm{~K}$ ). Thus the effects of $h_{x}$ on $S^{x x}(q, \omega)$ and $S^{y y}(q, \omega)$ discussed here are yet to be observed in a real system.

Apart from the relevance to experimental systems such as $\mathrm{CsCoCl}_{3}$ and $\mathrm{CsCoBr}_{3}$, the present study is intended to provide insights into the spin dynamics of the fully anisotropic Ising-like AFM system in the presence of a transverse magnetic field $h_{x}$. The ground-state energy and low-lying excitation spectrum of the fully anisotropic Hamiltonian are known exactly because of the mapping between the fully anisotropic Hamiltonian and the exactly solvable eight-vertex model $[15,16]$. Our calculations provide us with some physical insights into the spin dynamics in an Ising-like fully anisotropic AFM system in the presence of a transverse magnetic field.

## References

[1] Villain J 1975 Physica B 791
[2] Ishimura N and Shiba H 1980 Prog. Theor. Phys. 63745
[3] Hirakawa K and Yoshizawa H 1979 J. Phys. Soc. Japan 46455
[4] Yoshizawa H, Hirakawa K, Satija S K and Shirane G 1981 Phys. Rev. B 232298
[5] Boucher J P, Regnault L P, Rossat-Mignod J, Henry Y, Bouillot J and Stirling W G 1985 Phys. Rev. B 313015
[6] Nagler S E, Buyers W J L, Armstrong R L and Briat B 1983 Phys. Rev. B 271784
Nagler S E, Buyers W J L, Armstrong R L and Briat B 1983 Phys. Rev. B 283873
[7] Lehmann W P, Breitling W and Weber R 1981 J. Phys. C: Solid State Phys. 144655
[8] Matsubara F and Inawashiro S 1991 Phys. Rev. B 43796
[9] Murao K, Matsubara F and Inawashiro S 1995 J. Phys. Soc. Japan 64275
[10] Goff J P, Tennant D A and Nagler S E 1995 Phys. Rev. B 5215992
[11] Bose I and Ghosh A 1996 J. Phys.: Condens. Matter 8351
[12] Yoshizawa H, Hirakawa K, Satija S K and Shirane G 1981 Phys. Rev. B 232298
[13] Shiba H 1980 Prog. Theor. Phys. 64466
[14] Mekata M and Adachi K 1978 J. Phys. Soc. Japan 44806
[15] Johnson J D, Krinsky S and McCoy B M 1973 Phys. Rev. A 82526
[16] Baxter R J 1972 Ann. Phys., NY 70323

